A Numerical Model and Spreadsheet Interface for Pumping Test Analysis

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Abstract

Curve-matching techniques have been the standard method of aquifer test analysis for several decades. A variety of techniques provide the capability of evaluating test data from confined, unconfined, leaky aquitard, and other conditions. Each technique, however, is accompanied by a set of assumptions, and evaluation of a combination of conditions can be complicated or impossible due to intractable mathematics or nonuniqueness of the solution.

Numerical modeling of pumping tests provides two major advantages: (1) the user can choose which properties to calibrate and what assumptions to make; and (2) in the calibration process the user is gaining insights into the conceptual model of the flow system and uncertainties in the analysis. Routine numerical modeling of pumping tests is now practical due to computer hardware and software advances of the last decade.

The RADFLOW model and spreadsheet interface presented in this paper is an easy-to-use numerical model for estimation of aquifer properties from pumping test data. Layered conceptual models and their properties are evaluated in a trial-and-error estimation procedure. The RADFLOW model can treat most combinations of confined, unconfined, leaky aquitard, partial penetration, and borehole storage conditions. RADFLOW is especially useful in stratified aquifer systems with no identifiable lateral boundaries. It has been verified to several analytical solutions and has been applied in the Snake River Plain Aquifer to develop and test conceptual models and provide estimates of aquifer properties. Because the model assumes axially symmetrical flow, it is limited to representing multiple aquifer layers that are laterally continuous.

Introduction

Pumping tests are among the most frequently used methods of estimating aquifer properties. Most commonly, curve-matching techniques are applied either graphically or with computer-based optimization routines to achieve an adequate or possibly optimal match between theoretical relationships and measured field data. Curve matching has been the traditional means of evaluating pumping test data for several decades. A variety of type curves and procedures are available to fit an assortment of different field conditions (Lohman 1979). The Theis curve is applicable to conditions that can be represented by a perfectly confined, homogeneous, and isotropic aquifer of infinite areal extent pumped by a fully penetrating well of infinitely small diameter. One or several of these accompanying assumptions are relaxed or removed in the assortment of more recently developed analytical techniques. Some of the more commonly applied analytical techniques address leaky aquitard conditions (Hantush and Jacob 1955; Hantush 1960), unconfined aquifers (Neuman 1974; Streltsova 1974; Moench 1996; Moench 1997), and partially penetrating pumping wells (Papadopulos and Cooper 1973; Neuman 1974). Because of the wide array of techniques available, users are often able to find a method that adequately approximates field conditions at a given location. Use of any of the methods, however, carries with it a prescribed list of assumptions that the user must hope, often optimistically, are not significantly violated in the application of interest. In some cases, it is also possible that a user may be unaware of accompanying assumptions and their significance to the analysis.

Computerized curve-matching programs have streamlined the process and improved accuracy but adhere to the same theoretical assumptions and restrictions. Computer techniques have also automated the process to a point where users need minimal experience and understanding of ground water flow systems. Unfamiliarity with the techniques and accompanying assumptions may lead to inappropriate application of methods and misleading conclusions.

Numerical modeling capabilities for pumping test analysis have been available for more than 20 years. Rushton and Booth (1976), Lakshminarayana and Rajagopalan (1977, 1978), Rathod and Rushton (1984, 1991), Bennett et al. (1990), Butler and McElwee (1990), Rutledge (1991), Reilly and Harbaugh (1993), and Pandit and Aoun (1994), among others, have applied, developed, or promoted the use of numerical models for pumping test analysis. Finite difference and finite element codes have been developed for pumping test analysis in one, two, and three dimensions. The popular MODFLOW code has been adapted to include a solution in cylindrical coordinates specifically to evaluate drawdown around a well (Reilly and Harbaugh 1993). Numerical models can be created that require few of the assumptions that accompany analytical techniques. In many cases, the user can tailor the list of assumptions to meet the needs of each specific situation. In addition, trial-and-

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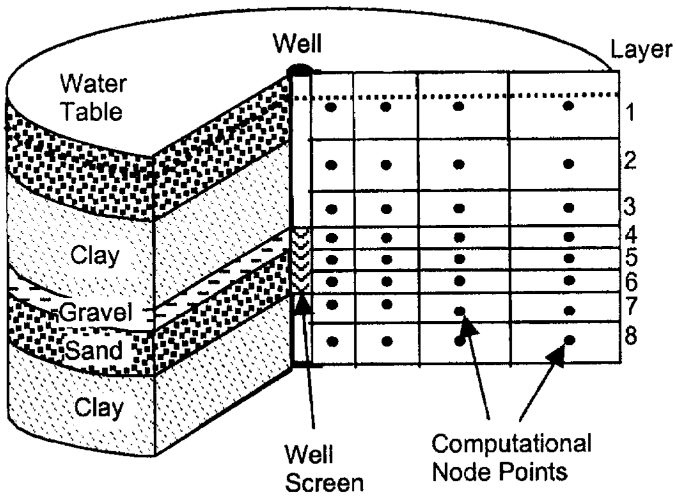


Figure 1. Model grid and conceptual representation.

error calibration to pumping test data, with properly presented results, may provide the user with more insights into the conceptual nature of the flow system than would be achieved with analytical techniques. Some authors have promoted sensitivity analysis with numerical models as an integral part of pumping test analysis (Butler and McElwee 1990; Jiao 1995). Computer-based analytical techniques are often equipped with sensitivity analysis features as well.

Despite the potential advantages of numerical models for pumping test analysis, most practitioners appear to favor curve-matching techniques. The lack of widespread acceptance of numerical models may be due to a lack of convenient input and output interfaces and the amount of time required to gain facility with earlier models. Setting up input data sets, populating with appropriate data, and processing results for comparison to observation well data is certainly more complicated than selecting and applying a computerized curve-matching technique. With modern computer hardware and software, however, numerical modeling can be performed nearly as easily as curve matching.

The purpose of this paper is to demonstrate the utility of numerical modeling for routine analysis of pumping test data through the use of RADFLOW, a two-dimensional, finite-difference model that simulates axially symmetrical flow to a well. The RADFLOW model places a greater emphasis on automating modeling procedure and minimizing the need for modeling expertise by the user than many previous works. The model is operated from an EXCEL spreadsheet interface, specifically designed for analysis of pumping test data. Model execution, input preparation, and output display are integrated into the spreadsheet. Users can quickly set up and graphically compare simulation results and field-collected time-drawdown data. The assumption of axially symmetrical flow limits representation of lateral heterogeneities, but few other assumptions are imposed on users. Users can develop and represent their own conceptual model of the system rather than conform to one of several prescribed conceptual models associated with analytical techniques. A mixture of partial penetration, one or more leaky aquitards, unconfined conditions, and borehole storage concepts may be combined in a single simulation. Unlike analytical techniques, the layered representation in RADFLOW imposes no assumptions of vertical flow or storage in aquitards, horizontal flow in aquifers, infinitesimally small well diameter, time limitations on solution validity or other common constraints. RADFLOW provides the opportunity for hydrogeologists to make full use of their subjective understanding of the aquifer conceptual model in combination with the quantitative information of pumping tests.

Modeling Axially Symmetrical Flow

Pumping test analytical techniques vary in complexity. With increasing degrees of complexity, fewer assumptions are imposed and a larger number of aquifer characteristics are estimated at the cost of increasing nonuniqueness of estimates. Numerical models designed for pumping test analysis may also be designed with varying degrees of complexity. The goal of this paper is to present a model that is applicable to a large number of field situations, but is not excessively complicated or cumbersome, and does not lead the user to create high degrees of nonuniqueness in calibration.

The RADFLOW model simulates two-dimensional, axially symmetrical flow to a well (i.e., radial and vertical components). The model grid consists of a series of stacked cylinders, concentric about the center of the pumping well (Figure 1). This representation inherently creates the assumption that a lateral variation in aquifer properties occurs in a radially symmetric pattern around the pumping well. Due to the implausibility of axially symmetrical heterogeneity in a natural system, the input to RADFLOW has been simplified by limiting the model to simulation of multiple aquifer layers that are homogeneous within each layer (Figure 1).

Many aquifer test situations can be reasonably approximated as a series of homogeneous layers. The greatest contrast in aquifer properties often occurs between layers, as opposed to laterally within the system. Often the best definition of aquifer variability is provided in the vertical dimension by interpretation of drillers' and geophysical logs of the pumping and observation wells. Seldom is knowledge of geologic variation in the horizontal dimension sufficient to guide estimation of the horizontal distribution of properties in a pumping test analysis. Analytical techniques also generally require strict adherence to this assumption. Limiting the simulation to a sequence of laterally homogeneous layers also restricts the number of calibration parameters, constraining the nonuniqueness of the results. As with most analytical techniques, RADFLOW users should evaluate available geologic information to determine the potential effects of lateral heterogeneity and anisotropy on test results.

RADFLOW is a finite-difference model. The finite-difference expressions representing flow between grid cylinders are written in terms of radial distance from the center of the pumping well (r) and vertical distance above a reference datum (z). Horizontal and vertical flow components (Q) to and from each model node are computed through application of Darcy's law in cylindrical coordinates:

$$Q = -(KA / \Delta l)\Delta h \qquad (1)$$

The terms enclosed in parentheses are collectively termed the hydraulic conductance, K is hydraulic conductivity, A is the cross-sectional area, Δl is the node separation distance radially or vertically, and Δh is the head difference between corresponding node points. Calculation of the cross-sectional area and the node separation distance is different between radial flow and rectangular coordinate based models. Reilly and Harbaugh (1993) provide an excellent description of the computations. The following discussion

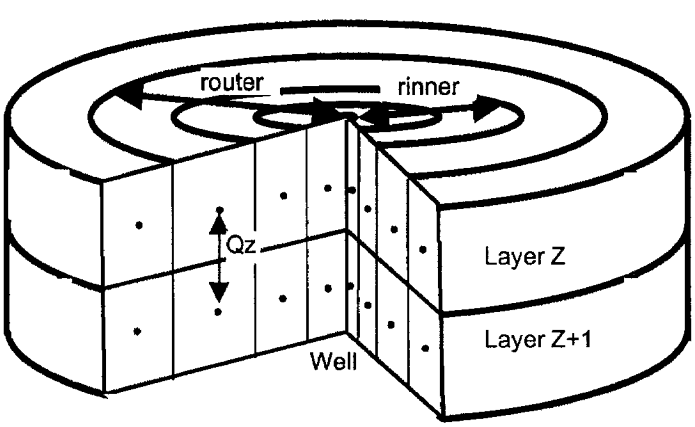


Figure 2. Surface area representation for vertical flow computation.

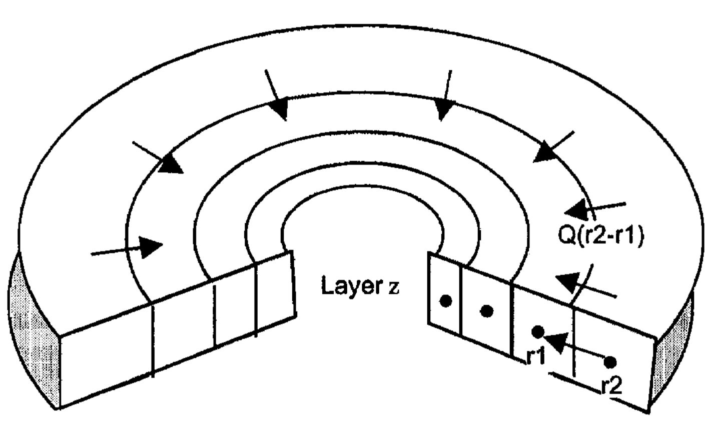


Figure 3. Representation of the radial flow component.

is similar to that of Reilly and Harbaugh (1993) with some adaptation to the unique characteristics of RADFLOW.

When determining vertical hydraulic conductance between model nodes, the cross-sectional area must be adapted to reflect flow through the end of a cylindrical section. The cross-sectional area (A) is computed as $\pi(r_{outer}^2 - r_{inner}^2)$, where r_{inner} and r_{outer} represent cell boundaries in the radial direction and are equivalent to half the distance between the computational node and inner and outer adjacent nodes, respectively (Figure 2). Estimates of vertical and lateral hydraulic conductivity of each layer are entered by the user. The computational node points are vertically centered within each layer (Figure 1). The effective hydraulic conductivity used in the computation of vertical flow between cells is the harmonic mean of the vertical hydraulic conductivities of the two layers. This follows the standard practice of block-centered model grids.

The radial flow component requires a different computation of the variables in Equation 1 (Figure 3). The cross-sectional area through which flow is occurring is increasing with increasing distance from the pumping well. The expression of Darcy's law must therefore be integrated across the radial dimension of the cell to properly represent converging flow:

$$Q = K\,(2\pi r\Delta z)dh / dr \text{ or } dr / r = (K / Q)(2\pi\Delta z)dh \qquad (2)$$

Integrating the second equation with respect to r and h, between the limits of r_1 and r_2 (radii of nodes between which flow is being computed) and the corresponding hydraulic head values h_1 and h_2:

$$\int_{r2}^{r1} (1 / r)\,dr = \int_{h2}^{h1} (2\,\pi\Delta\,zK / Q)dh \qquad (3)$$

results in

$$\ln(r_1 / r_2) = (2\,\pi\Delta zK / Q)(h_1 - h_2) \qquad (4)$$

rearranging terms:

$$Q(r_2 \rightarrow r_1) = (2\pi\Delta zK)(h_1 - h_2) / \ln(r_1 / r_2) \qquad (5)$$

Where Q is the flow rate and is positive for radially inward flow, Δz is the layer thickness, and dh/dr is the hydraulic gradient in the radial direction. In this case, hydraulic conductance (Cr) is represented as

$$Cr = (2\,\pi K\,\Delta z) / \ln(r_1 / r_2) \qquad (6)$$

The series of simultaneous equations established for the two-dimensional finite-difference grid are solved for each time step in RADFLOW by an alternating-direction implicit method modified from the model developed by Prickett and Lonnquist (1971). The computed head distribution at each model time step is built from the head distribution of the previous time step by a backward-difference solution technique. The method is similar to that of the U.S. Geological Survey MODFLOW model (McDonald and Harbaugh 1988).

The initial head distribution is uniform and established by user input. This initial value is used to establish the position of the water table and an initial value of head at each model node. Since the user is calibrating to drawdown, rather than aquifer head, the assumption of a uniform distribution of starting head is generally acceptable so long as the position of the water table is appropriately represented. This assumption may not hold if nonlinear relationships are significant due to dewatering of an unconfined aquifer. Effects of transient external influences, such as nearby pumping or barometric pressure variations, are a concern in many pumping tests. The drawdown data should be adjusted to compensate for these external effects prior to the RADFLOW calibration.

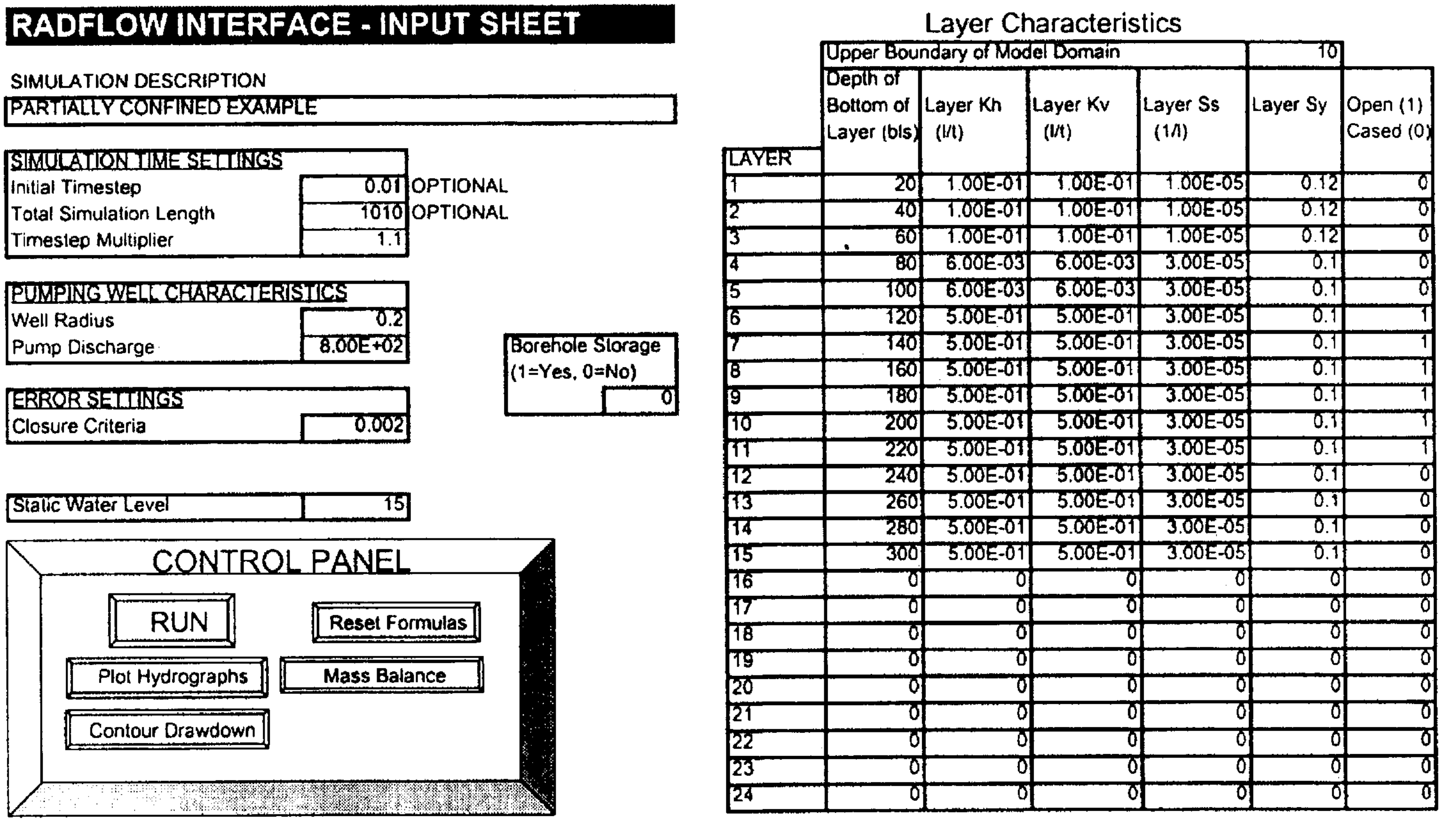
Boundaries of the RADFLOW model domain are established as follows.

Outer Boundary — The model is intended to represent a laterally infinite aquifer. A no-flow boundary is automatically set at a distance sufficient to produce negligible error in the simulation results. The distance is automatically determined as a modification of a convention employed by Pandit and Aoun (1994) where the radius of influence (R) is estimated as a function of aquifer transmissivity (T), elapsed pumping time (t), and aquifer storativity (S):

$$R = 2(2.25\ Tt / S)^{0.5} \qquad (7)$$

The Pandit and Aoun (1994) equation was modified by multiplying by a factor of two because verification tests indicated boundary effects may appear at lesser distances. The model grid is established to extend from the pumping well radius to beyond the radius of influence (see grid discretization discussion). Consequently, the grid changes each time the user adjusts values of aquifer properties or simulation length. In cases where observation wells are at large distances from the pumping well and extreme accuracy is required, the outer boundary may need to be extended. This can be accomplished by extending the simulation period beyond that of the pumping test.

Inner Boundary — A mixed specified-flux and no-flow boundary is established at the user-prescribed well radius. The



RADFLOW INTERFACE - INPUT SHEET

SIMULATION DESCRIPTION

PARTIALLY CONFINED EXAMPLE

SIMULATION TIME SETTINGS

Initial Timestep	0.01	OPTIONAL
Total Simulation Length	1010	OPTIONAL
Timestep Multiplier	1.1	

PUMPING WELL CHARACTERISTICS

Well Radius	0.2
Pump Discharge	8.00E+02

Borehole Storage (1=Yes, 0=No) 0

ERROR SETTINGS

Closure Criteria	0.002

Static Water Level	15

CONTROL PANEL

RUN

Reset Formulas

Plot Hydrographs

Mass Balance

Contour Drawdown

Layer Characteristics

Upper Boundary of Model Domain 10

LAYER	Depth of Bottom of Layer (bls)	Layer Kh (l/t)	Layer Kv (l/t)	Layer Ss (1/l)	Layer Sy	Open (1) Cased (0)
1	20	1.00E-01	1.00E-01	1.00E-05	0.12	0
2	40	1.00E-01	1.00E-01	1.00E-05	0.12	0
3	60	1.00E-01	1.00E-01	1.00E-05	0.12	0
4	80	6.00E-03	6.00E-03	3.00E-05	0.1	0
5	100	6.00E-03	6.00E-03	3.00E-05	0.1	0
6	120	5.00E-01	5.00E-01	3.00E-05	0.1	1
7	140	5.00E-01	5.00E-01	3.00E-05	0.1	1
8	160	5.00E-01	5.00E-01	3.00E-05	0.1	1
9	180	5.00E-01	5.00E-01	3.00E-05	0.1	1
10	200	5.00E-01	5.00E-01	3.00E-05	0.1	1
11	220	5.00E-01	5.00E-01	3.00E-05	0.1	1
12	240	5.00E-01	5.00E-01	3.00E-05	0.1	0
13	260	5.00E-01	5.00E-01	3.00E-05	0.1	0
14	280	5.00E-01	5.00E-01	3.00E-05	0.1	0
15	300	5.00E-01	5.00E-01	3.00E-05	0.1	0
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	0	0	0
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0

Figure 4. Example of RADFLOW interface input screen.

boundary is fixed-flux at locations identified by the user as corresponding to depths where the well is open to the aquifer (e.g., screened or uncased). All other locations on the inner boundary are no-flow based on the concept of radially symmetrical flow toward the well. Borehole storage can optionally be included, which causes the inner boundary to be represented by the center of the well, and a storativity of unity is applied within the borehole.

Lower Boundary — A no-flow boundary is automatically assigned at the bottom of the user-defined domain. In the case of an extremely thick aquifer, the bottom may be set to an arbitrarily large depth to minimize effects on the simulation.

Upper Boundary — The upper boundary is simulated as a moving phreatic surface. The phreatic surface is allowed to move into deeper layers as dewatering occurs. Confined, unconfined, and leaky conditions can be simulated by varying vertical hydraulic conductivity of layers between the screened interval and the phreatic surface. In model cells containing the phreatic surface, the horizontal hydraulic conductance (Equation 6) is reduced in proportion to the thickness of the cell that is saturated. If dewatering occurs next to the screened portion of the well, then well discharge is redistributed among cells that represent the remainder of the open well interval. The adjustment to hydraulic conductance is made at the conclusion of each time step. This treatment appears adequate provided the entire aquifer thickness in which the pumping well is screened is not dewatered by the simulated pumping. Since the model is designed to represent only pumping conditions, no provisions are made for a rising water table.

Model Input

All user interaction is performed in the RADFLOW spreadsheet. Model input is performed in a spreadsheet template and includes field measurements of time-drawdown data, estimates of aquifer properties by layer, well construction characteristics, and time-constant pumping rate (Figure 4). Simulation constants are automatically set to default values and include the grid discretization, time-step multiplier, and simulation closure criteria. The user may optionally override the defaults by entering values in the appropriate fields. The calculation of default values is discussed in the section "Computation of Simulation Constants." The spreadsheet contains macros that (1) format the user-provided data for input to the Fortran program, (2) run the Fortran simulation code, and (3) direct simulation output back to the spreadsheet for graphical and statistical comparison to measured time-drawdown data.

Time and drawdown measurements are required for calibration of the aquifer properties. Measurements from as many as five observation wells (potentially including the pumping well) are entered or copied into a prescribed location in the spreadsheet. Additional observation wells can be accommodated as described in the user's manual (Johnson and Cosgrove 2000). Time and drawdown units must be consistent with units of all other input parameters. Radial distance from the pumping well and the depth to the center of the open interval of the observation well are also required inputs. It may be desirable to represent an observation well with a large open interval as multiple piezometers terminating in different layers in order to determine an average drawdown over the depth of open interval.

The conceptual model of the pumped aquifer may consist of as few as one or as many as 24 layers (Figure 4). The user is tasked with identifying the hydrologically significant layers and estimating properties of each layer. As a rule of thumb, thickness of adjacent model layers probably should not vary by a factor greater than 1.5 to control numerical error (Anderson and Woessner 1992).

RADFLOW restricts variation of the thickness of adjacent model layers to a factor of 2. Thick hydrologic layers should normally be represented as multiple model layers of identical properties in order to maintain a grid vertical expansion factor of less than 1.5. It is often convenient to use a fixed model layer thickness, such as 2 m, and use as many model layers as necessary to represent an aquifer or aquitard. Each layer used requires input of the depth to the bottom of the layer (normally relative to land surface), estimates of horizontal and vertical hydraulic conductivity, specific storage, and specific yield. Specific yield is required for all layers due to the potential for movement of the phreatic surface. Properties are assumed uniform throughout each layer and units must be consistent with all other data entry.

Pumping well construction characteristics are needed to identify the depths from which water is being pumped and for the optional computations of borehole storage. Pumping well radius and identification of the aquifer layers open to the well are required inputs. The time-constant pumping rate is also a required input and must be in units consistent with other inputs. Specific intervals from which pumping occurs may be specified by setting toggles associated with each identified model layer.

Input instructions are provided through pop-up comments that appear as the user moves the cursor to an input field. The instructions describe the required input and the basis of a default value if one is provided. Data checking is provided in the spreadsheet to ensure that the user does not accidentally enter physically unrealistic values, such as a specific yield greater than one.

Computation of Simulation Constants

The spreadsheet interface and the Fortran model are designed to minimize the need for modeling experience and to maximize user convenience. Simulation characteristics such as grid and time discretization are automatically calculated by the spreadsheet interface. The user is able to override some of the computed default values, thereby allowing greater flexibility in the model application. Computations of the default characteristics performed by the spreadsheet are described below. Some of the methods used to calculate the default values are similar to those used by Rathod and Rushton (1984), Pandit and Aoun (1994), and others. These methods have been combined and modified in RADFLOW to provide a simple model that performs adequately under a wide range of conditions.

Pumping Rate — The pumping rate is distributed among layers identified as open to the pumping well. RADFLOW automatically distributes well discharge among screened layers according to the product of the hydraulic conductivity and thickness of the layer. Layers above the phreatic surface are incapable of yielding water.

Time Discretization — Default values of the duration of the simulation and the initial time-step length are determined from user-entered time-drawdown measurements. The initial time step is set to be an order of magnitude smaller than the time of the first observation. Total simulation time is set as 1% larger than the time of the last observation (relative to start of pumping) entered in the measured time-drawdown data. Successive time steps increase by a user-selected factor (i.e., time-step multiplier) between 1 and 1.5. A progressive increase in time-step length is often suitable for pumping test analysis where aquifer water levels decline most rapidly at the beginning of the test. The authors have found that a time-step multiplier of 1.5 works satisfactorily in most applications

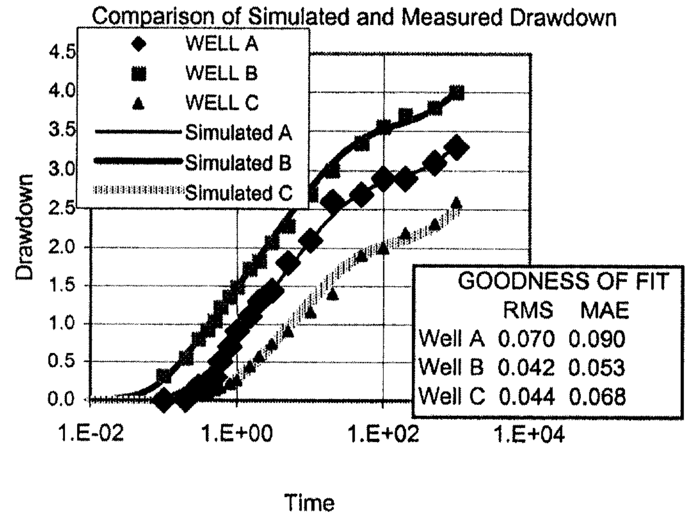


Figure 5. Example of time-drawdown comparison provided by the RADFLOW interface.

and minimizes computation time and computer storage requirements. A smaller multiplier may be useful in some applications to improve solution convergence or refine time-step interpolation. Time steps do not need to match the individual observation data. The user is allowed to override the initial time-step length and the total duration of the simulation. This will allow the user to view simulation results beyond the duration of the pumping test.

Grid Discretization —The radial grid is automatically developed based upon input well radius and a computed outer boundary location that is sufficiently distant to not introduce significant error into the analysis due to boundary effects. The innermost grid interval is established as the well radius (provided borehole storage effects are not being evaluated). The increment continuously increases by a factor of 1.5 until the radius of influence (Equation 7) is exceeded. The radial grid is automatically computed to minimize user effort and the need for modeling expertise. The grid is displayed but cannot be changed by the user (other than by changing simulation time or aquifer properties). The vertical grid is established to match layer specifications provided by the user.

Borehole Storage — Borehole storage is optionally simulated by a toggle switch provided on the spreadsheet. When borehole storage is activated, the model automatically assigns a specific yield of 1.0 to the water surface in the borehole. Vertical and horizontal hydraulic conductivities of the interior of the borehole are set to a value five orders of magnitude greater than that of the surrounding aquifer. This allows nearly unimpeded flow to be simulated within the borehole.

Model Output

Model output is routed back to the spreadsheet interface through macros assigned to spreadsheet buttons (Figure 4). The output consists of (1) simulated time-drawdown data for locations corresponding to observation wells, (2) the drawdown distribution at the end of the simulation, and (3) the mass-balance discrepancy at each time step. Each of the outputs is automatically displayed in a graphical format.

Simulated time-drawdown data at cells radially bordering the observation wells are imported to the spreadsheet from the Fortran program. The spreadsheet linearly interpolates drawdown for the

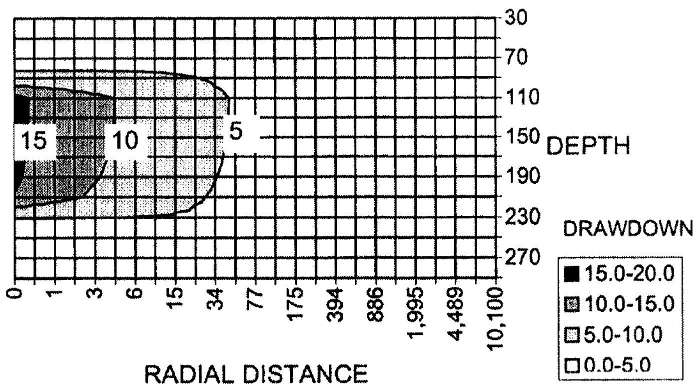


Figure 6. Example of cross-sectional drawdown contours (representing the end of the simulation period) provided by the RADFLOW interface.

radius of the observation wells (between two adjacent model nodes) and plots the simulated drawdown on a time-drawdown graph along with the measured values (Figure 5). The user attempts to match simulated and measured values as closely as possible by altering layer characteristics. One of the advantages of using a spreadsheet interface is the flexibility it provides the user in displaying results in a variety of formats. Although the default spreadsheet characteristics include a semilog time-drawdown graph, users familiar with spreadsheets can easily alter the setting to make the graph arithmetic, log-log, or other variations.

A linear temporal interpolation of drawdown is also performed to estimate drawdown at times corresponding to individual measurements of the observation wells. These values are used to compute the mean absolute error (MAE) and root mean squared (RMS) of residuals between simulated and measured drawdowns for each well. The statistics are intended to provide the user with a quantitative evaluation of the goodness of fit between simulated and measured drawdown to assist in calibration. In all cases evaluated, the use of a linear interpolation in the time and radial dimensions did not introduce significant error into the comparison of measured and simulated drawdowns.

RADFLOW includes the capability to view the vertical distribution of drawdown at a time corresponding to the end of the simulation period (Figure 6). Drawdown is usually contoured in the horizontal plane, indicating the areal spread of a cone of depression. In the case of the RADFLOW output, the contours are prepared in the vertical plane showing the cross-sectional propagation of drawdown. A contour plot along the horizontal plane would show only concentric lines of equipotential since axial symmetry is assumed. The spreadsheet three-dimensional graphic capabilities are limited in their ability to represent irregularly spaced data on the horizontal axis. This causes scale distortion of RADFLOW contour plots. The output, however, is still a useful presentation for understanding the propagation of drawdown throughout the aquifer cross section. This graph aptly demonstrates the controlling characteristics of aquifer properties on the distribution of drawdown in the vertical profile. The contours presented in Figure 6 correspond to the simulation conditions specified in Figure 4.

Model Calibration

Aquifer properties of user-designated layers are estimated through a trial-and-error calibration process, comparing simulated drawdown at selected locations to drawdown measured in a pumping test. This process provides the added benefit of providing an informal sensitivity analysis that educates the user on the nonuniqueness of model properties and sensitivity of estimated drawdown to specified characteristics. An understanding of sensitivity has been advocated as an important aspect of aquifer evaluation (Butler and McElwee 1990; Jiao 1995). Ideally, the hydrogeologist is as interested in improving the understanding of the conceptual model as in estimating aquifer properties.

It is suggested that users normally begin by representing a highly simplified concept of the aquifer system such as a one- or two-layer conceptual model (Frederick and Johnson 1996). Calibration of the single-layer model can be followed by a refinement of the vertical dimension into additional hydrologic layers and subsequent calibration. This incremental approach assists the user in understanding the benefits gained, in terms of improved fit, by each level of complication added to the conceptual model. Ideally, the simplest conceptual model that provides a reasonable fit to the measured data is desired.

Parameter estimation techniques can be applied with RADFLOW. These techniques provide two basic advantages: (1) determination of optimum estimates of aquifer properties within the constraints specified by the user, and (2) calculation of the correlation of estimated parameters. The parameter correlation indicates the degree of nonuniqueness in the solution. For example, a high degree of correlation between the vertical hydraulic conductivity of one layer, and the horizontal hydraulic conductivity of an overlying layer would indicate that nearly as good of a match between simulated and measured data could have been achieved with compensating changes in these two properties. The two primary disadvantages of using RADFLOW with parameter estimation routines are: (1) the additional complexity that is involved in setting up the simulations and interpreting the results, and (2) the lack of conceptual understanding that is gained from trial-and-error calibration. Although parameter estimation routines can be coupled with RADFLOW, at present the process is cumbersome. Input and output file formats for the Fortran program are described in the user's manual for those wanting to couple RADFLOW with parameter estimation routines.

Nonuniqueness of parameter estimates is an issue with application of both analytical techniques and numerical modeling. The nonuniqueness of parameter estimates generally increases with the number of parameters being estimated. Application of numerical models to pumping test analysis creates the opportunity to estimate large numbers of parameters when multiple aquifer layers are represented. Users are therefore encouraged to represent their conceptual model of the real system with as simple a numerical model as possible, and to collect as much test data at different depths and distances as possible. It is worth noting that fewer parameters may be estimated using analytical techniques because some aquifer properties have been assigned prescribed values. For example, when using the Theis method, it is assumed that the hydraulic conductivity of overlying and underlying materials is zero. Application of the Theis method may yield a more unique solution than a numerical method that requires estimation of aquitard characteristics; however, that solution is constrained by an assumption that may not be representative of the physical system. Use of a numerical model such as RADFLOW allows the user to represent more parameters, constraining some of the parameter values based on subjective information or professional judgment.

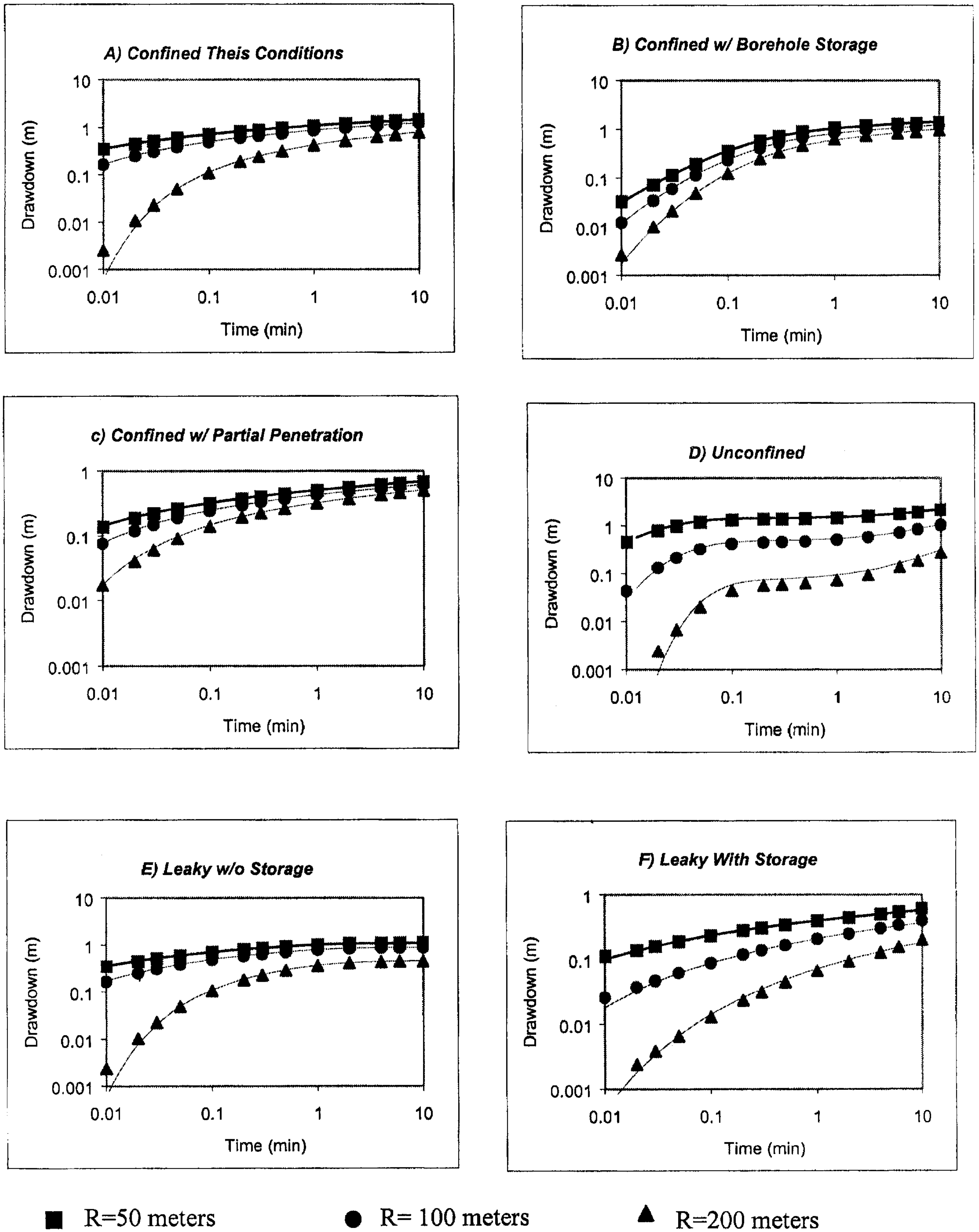


Figure 7. Comparison of analytical solutions to simulation results (simulation results are presented as symbols, analytical solutions as lines).

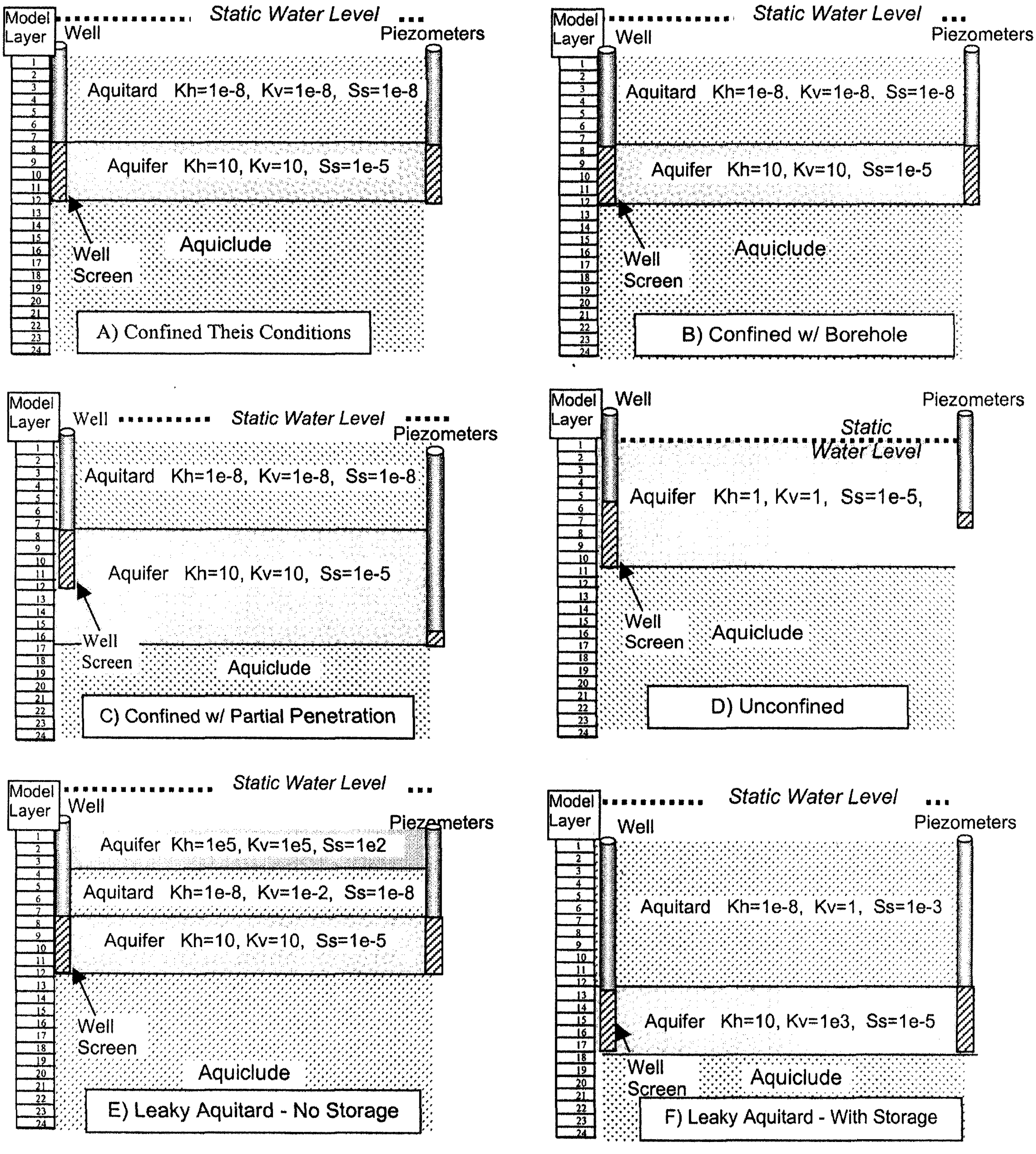


Figure 8. Input illustrations for selected verification simulations.

Model Validation

The RADFLOW model was validated by comparison to a selected set of analytical solutions. Simulations were performed to represent ideal confined conditions with partially penetrating wells, ideal confined conditions with borehole storage, leaky aquitard conditions, and unconfined conditions. A comparison of simulated and analytically determined drawdown is presented in Figure 7 for observation wells located at 50, 100, and 200 m from a pumping well discharging at a rate of 1000 m^3/day. Default time and grid discretization, computed by RADFLOW, were used for each of the simulations. Time steps were set to progressively increase by a factor of 1.5 (time step multiplier). The duration of the simulations varied, depending on the time scale of data entered for verification. The radial extent of the grid automatically varied, depending on the radius of influence, as calculated by Equation 7. Vertical discretization of the grid was developed such that each lithologic layer was composed of at least two model layers, and adjacent model layer thickness varied by a factor no greater than 1.5. Each of the above simulations is described in the following paragraphs to provide the reader with an understanding of how different conditions are represented.

Ideal confined aquifer conditions (Theis 1935) are represented in a two-layer system with the phreatic surface located in or above the upper confining layer (Figures 7a and 8a). The pumping well is completed throughout the thickness of the aquifer. Vertical hydraulic conductivity of the aquifer is immaterial since water is withdrawn uniformly along the length of the fully penetrating borehole, producing horizontal flow. Vertical and lateral hydraulic conductivity and specific storage of the upper, nonpumped layer (aquitard) are set several orders of magnitude below the values representing the aquifer to produce negligible leakage. Borehole storage effects can be included without changing the model representation except for switching a toggle on the input spreadsheet. The graphical comparison of drawdown for borehole storage conditions (Figure 7b) is based on aquifer conditions identical to those shown for the Theis simulation (Figure 8b) with a borehole diameter of 5 m (sufficiently large to display borehole storage effects). Simulation results are nearly identical to the analytical solution for large boreholes presented by Papadopulos and Cooper (1973). At the 200 m distance, the first data point appears to depart slightly from the analytical results (0.006 m). This difference is not unusual, but it is apparent because of the log scale of the vertical axis. The difference is primarily the result of numerical and discretization error. The same is true for several of the validation simulations.

Partial Penetration conditions are simulated by terminating the last open interval of the pumping well above the bottom of the aquifer (Figures 7c and 8c). A thicker aquifer unit is represented to make partial penetration effects more prominent. Observation wells are completed 45 m below the bottom of the pumping well that is completed in the upper half of the aquifer. Simulation results compare favorably with the analytical solution of Hantush (1961a, 1961b).

Unconfined, delayed response is represented in the example with an isotropic aquifer and a static water level located near the top of the aquifer (Figures 7d and 8d). The pumping well is screened in the lower half of the aquifer. Aquifer thickness is sufficiently large (100 m) to avoid significant changes in transmissivity due to dewatering. Differences between simulated and analytical solutions (Neuman 1974) are apparent at the 200 m distance. Additional simulations indicate that these differences are primarily the result of two conditions: (1) the analytical technique assumes a constant aquifer thickness, and (2) the layer discretization in RADFLOW (10 m layers). Use of 5 m thick layers improves the match with the analytical technique.

Leaky aquitards, with and without storage, overlying pumped aquifers are simulated according to the conceptual models presented in Figures 8e and 8f. Figure 8e represents conditions in which the aquitard has negligible storage capabilities. Simulation results compare favorably to the analytical solution of Hantush and Jacob (1955) in Figure 7e. Figure 8f represents a situation in which the aquitard has significant potential to release water from storage. Simulation results in Figure 7f match the analytical solution of Hantush (1960) reasonably well. In both of the leaky aquitard simulations some early time departure is apparent.

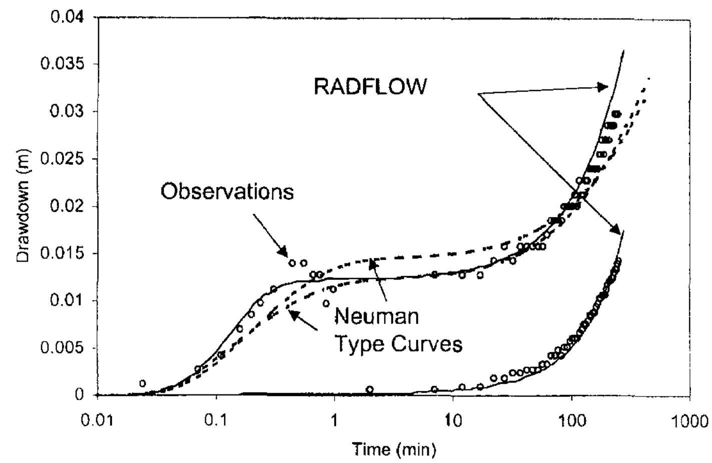


Figure 9. Comparison of time-drawdown data from the Snake River Plain Aquifer pumping test to RADFLOW simulation results and a selected analytical technique.

In general, the simulation results compare well to analytical results. Small departures result from numerical error, discretization error, and interpolation error. Numerical error occurs with all numerical models and results from iterative approximation of the true solution. Numerical error can be controlled by reducing the closure criterion; however, an excessively small criterion may result in large simulation times or failure of a model to converge. RADFLOW compares the user-entered closure criterion against the absolute value of iteration residuals summed for all cells. A closure criterion one order of magnitude less than what is considered an acceptable error is generally adequate. Discretization error results from averaging aquifer properties and hydraulic head within the volume of a cell. It is recommended that users represent each conceptual layer by multiple model layers to minimize discretization error. For the validation simulations, the largest mean absolute error occurred in the borehole storage simulation in which the difference between observed and simulated values averaged less than 5% of the analytically determined drawdown for points shown in Figure 7c. Although these differences are probably not a concern in most pumping test analyses, errors can be further reduced by a more refined layer discretization.

Example Application

A straddle-packer system was used to collect pumping test data from an observation well 1220 m away from a large capacity (11.4 m^3/min) production well at the Idaho National Engineering and Environmental Laboratory in southern Idaho. The wells are completed in the Snake River Plain Aquifer, hosted in unconfined layered basalts and interbedded sediments. Effective aquifer thickness is unknown, but it is generally believed to be in excess of 75 m in most locations (Robertson 1974; Garabedian 1992). The production and observation wells both penetrate a dominant sediment layer that is believed to be laterally continuous. The effect of the sediment layer on vertical flow is unknown. The production well is screened both above and below the sediment interbed. The straddle packer used pressure transducers to measure drawdown both above and below the sediment layer. The pumping test conceptual model is unconfined with possibly a leaky aquitard (sediment interbed) approximately 170 m below land surface. Production and observation wells are partially penetrating.

The conceptual model is difficult to represent with traditional analytical curve-matching techniques. Unconfined, partial penetration type curves developed by Neuman (1974) probably provide the best representation. These type curves include effects of anisotropy in the vertical plane. Time-drawdown data from the two observation well transducers are plotted with two corresponding Neuman type curves in Figure 9. An emphasis was given to matching the observations of the deeper transducer in this application, although the emphasis could just as easily have been on the shallow observations. In either case, it is not possible to satisfactorily match both data sets with a single set of aquifer properties because the conditions do not conform well to the assumptions of the technique. This is illustrated by the large departure between the type curve for the deeper zone and the observations in that zone (Figure 9).

A three-layer RADFLOW simulation was also calibrated to the observation well data. The conceptual model was composed from subsurface stratigraphy as identified from well and geophysical logs. Identical aquifer properties were assigned to model layers representing a single hydrogeologic unit to minimize estimation nonuniqueness. Trial-and-error calibration of aquifer and aquitard hydraulic conductivity (horizontal and vertical), specific storage, and specific yield produced results that compare favorably with observed drawdown (Figure 9). Parameter perturbations indicated that the simulated drawdown was most sensitive to specific yield and aquitard vertical hydraulic conductivity. Results were relatively insensitive to changes in aquitard horizontal hydraulic conductivity and specific storage, and upper aquifer specific storage. Consequently, values estimated for these characteristics are probably not meaningful. Similar parameter estimates were obtained using the PEST (Doherty 1998) parameter estimation code. The RADFLOW application to the Snake River Plain Aquifer permitted analysis of aquifer properties based on the hydrogeologist's conceptual model rather than forcing a fit to a misrepresentation provided by an analytical tool. More information on the pumping test and this application of RADFLOW can be found in Frederick and Johnson (1996).

Conclusions

Curve-matching analytical techniques have been the standard approach to aquifer test analysis for several decades. A variety of analytical techniques provide the capability of evaluating test data from confined, unconfined, leaky aquitard, partial penetration, and other conditions. Each technique, however, is accompanied by a set of assumptions, and evaluation of a combination of conditions can be complicated or impossible due to intractable mathematics or nonuniqueness of the solution.

Numerical modeling of pumping tests provides two major advantages: (1) the user can selectively choose which properties to calibrate and what assumptions to make; and (2) in the trial-and-error inverse solution process the user is performing an informal sensitivity analysis that provides insights into the conceptual model of the flow system and uncertainties in the analysis.

Routine numerical modeling of pumping tests is now practical due to computer hardware and software advances of the last decade that can expedite model development and output interpretation and help reduce the need for modeling expertise of the users. The RADFLOW model is designed specifically to meet these objectives.

The RADFLOW model presented in this paper is an example of an easy-to-use model and interface for trial-and-error estimation of aquifer properties from pumping test data. The RADFLOW model can treat most combinations of confined, unconfined, leaky aquitard, partial penetration, and borehole storage conditions. RADFLOW is especially useful in stratified aquifer systems with no identifiable lateral boundaries. It has been verified to several analytical solutions and has demonstrated advantages over analytical techniques in an application to the Snake River Plain Aquifer in Idaho. Because RADFLOW represents an axially symmetrical system, it is limited to uses where lateral heterogeneity and anisotropy in the horizontal plane may be neglected. A user's manual and the files that comprise the RADFLOW package can be downloaded free of charge from the Idaho Water Resources Research Institute Web site at http://teton.if.uidaho.edu/~ifiwrri/radflow.html.

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